Development of a Model of Cycling Unvented Fireplace Use

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ABSTRACT

Unvented heating appliances have become common in the United States. These appliances release all combustion products into the space in which they are located. The intended use according to manufacturers of these appliances is as supplemental heating devices. One common control strategy for this usage pattern is to use a thermostat to cycle the fireplace on and off. This means that the combustion product emissions will be intermittent in what will frequently be an approximately uniform pattern. To investigate the impact of various parameters on indoor pollutant concentrations resulting from this type of unvented fireplace use, it is useful to have a model that accounts for this type of cycling behavior. This paper presents the development of a model that assumes cycling pollutant emission but constant dilution

Practical Implications: *The model developed here provides a means for assessing the indoor contaminant concentrations resulting from an intermittent source such as a cycling unvented gas fireplace. The model is a closed-form solution, allowing the calculation of concentrations at any time based on assumptions of generation rate, air change rate, cycling time, and fractional on-time of the source. This allows for the determination of the most important factors influencing the resulting concentrations, and which contaminants are most likely to exceed levels of concern, such as health-based standards and guidelines from organizations such as the US EPA, the World Health Organization, etc.*

INTRODUCTION

The indoor concentration of any pollutant is a combination of generation and dilution. Previous work has resulted in a model for uniform generation but intermittent dilution (Sherman 2006). That is appropriate for a residential environment where there is constant emission via mechanisms such as respiration or off-gassing and ventilation fans are being used intermittently to remove these pollutants. However, for a source such as an unvented gas fireplace, which may be used intermittently, it is useful to have a model that reflects the intermittent usage and the dilution (often simply natural infiltration, though potentially continuous mechanical ventilation), which may be approximately constant over a period of interest. (In the event of a tight house with constant mechanical ventilation, the air change rate would be close to constant.) This paper presents the development of such a model. This development was part of a larger project that included in-situ measurement of combustion product concentrations in 30 homes in which unvented gas fireplaces were being used, as well as some limited laboratory studies and a national survey of unvented gas fireplace owners.

DeWerth et al. (1996) also have developed a model for cycling unvented gas fireplace use. Their model was essentially a time-series model, where the results at each time step were calculated based on the change from the previous time step. The model developed here has the advantage of being closed-form, thereby allowing for the calculation of gas concentration and room temperature at any time step as well as solving for other parameters analytically.

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MODEL DEVELOPMENT

Continuous Appliance Use

Though this paper is focused on cycling use of the fireplace, it is useful to look at the solution for the continuous case since these basic equations and identities will be used in the cycling case development.

Concentration. The change in concentration of a gas i with time (in minutes) can be expressed as

$$\frac{dC_{i,t}}{dt} = \frac{E_{i,f}q_f}{V} + \frac{E_{i,p}N_p}{V} + \frac{\mathrm{ACH}(C_{i,out} - C_{i,t})}{60} \quad (1)$$

where the first term on the right-hand side is the contribution (or reduction, in the case of oxygen) from the combustion appliance; the second term is the contribution (or reduction, in the case of oxygen) from the occupants of the building, and the third term is the change due to infiltration/ventilation, which can bring in some amount of the gas in question if there is any outdoors and which removes some amount of the gas from indoors.

For simplicity, ACH/60 can be replaced with κ_a and the first two right-hand-side terms can be replaced with the following identities:

$$K_{i,f} = \frac{E_{i,f}q_f}{V} \qquad K_{i,p} = \frac{E_{i,p}N_p}{V}$$

Using these identities, the solution for the concentration of gas i at time t is

$$\begin{aligned} C_{i,t} = {} & [K_{i,f} + K_{i,p} + \kappa_a C_{i,out} \\ & - (K_{i,f} + K_{i,p} + \kappa_a C_{i,out} - \kappa_a C_{i,0})\exp(-\kappa_a t)]/\kappa_a \end{aligned} \quad (2)$$

This equation allows for the evaluation of what the concentrations of combustion products will be at any particular time based on assumptions about the pertinent parameters, such as air change rate and emission rate of the combustion products. In addition, this equation provides the means to answer several other questions of potential interest in a closed-form manner, as described below. The equation can provide the long-term asymptotic (steady-state) concentration that would be reached if the appliance was operated continuously for a long time. The equation can provide the air change rate that would be required to avoid ever exceeding a concentration of interest regardless of how long the appliance is operated. It also can be used to calculate the amount of time that can elapse before a concentration of interest is exceeded. All of these applications provide the means of assessing which combustion products are most likely to become problematic first and what can be done to prevent problems from ever occurring. It must be cautioned, however, that the results are subject to assumptions such as emission rate, and to the extent that the assumptions are not representative of the conditions in a particular house, the results will also not be accurate for that house.

Asymptotic (Steady-State) Concentration. The steady-state concentration is determined by setting the time in the exponential in Equation 2 to infinity. The exponential then goes to zero, resulting in

$$C_{i,ss} = \frac{K_{i,f} + K_{i,p} + \kappa_a C_{i,out}}{\kappa_a} \quad (3)$$

Required Air Change Rate to Prevent Exceeding Threshold Concentration. The minimum air change rate required to avoid ever exceeding a threshold concentration $C_{i,crit}$ can be determined by solving for the air change rate in Equation 3. Recalling that κ_a = ACH/60, the minimum air change rate ACH_{crit} is given by

$$\mathrm{ACH}_{crit} = \frac{K_{i,f} + K_{i,p}}{C_{i,crit} - C_{i,out}} \times 60 \quad (4)$$

Time before Threshold Concentration Is Exceeded. Assuming that the air change rate of the building is not sufficient to keep the steady-state concentration below $C_{i,crit}$, Equation 2 can be rewritten to provide the amount of time that the appliance can be operated before $C_{i,crit}$ is exceeded, as follows:

$$\begin{aligned} t_{crit} = {} & [\ln(K_{i,f} + K_{i,p} + \kappa_a C_{i,out} - \kappa_a C_{i,crit}) \\ & - \ln(K_{i,f} + K_{i,p} + \kappa_a C_{i,out} - \kappa_a C_{i,0})]/-\kappa_a \end{aligned} \quad (5)$$

Room Temperature. Room temperature is important in modeling when evaluating relative humidity. For other gases, the room temperature is not important other than as a reality check for whether the modeled operation is reasonable. For example, it is unlikely that the appliance would be operated in a manner that would result in the indoor temperature exceeding 40ºC.

The temperature model that follows assumes that the indoor density is constant, despite the indoor temperature and humidity changing. While this is certainly not correct, the density does not change by more than a couple of percent over a range of likely indoor conditions. Attempting to account for both temperature and humidity impacts on indoor density renders the equation nonsolvable in closed form. On balance, we do not believe that the magnitude of the error resulting from assuming a constant indoor density justifies the much greater complexity of the model that would result from not making this assumption, especially as the model is intended to provide basic insight into the general level of combustion product accumulation. If the model were later to become part of a detailed hourly simulation that considered changing weather conditions and frequently changing appliance usage patterns, then it may be justified to develop a more complex model that would accommodate the change in indoor air density.

The heat balance in the space can be expressed as

$$c_a\rho_{in}V\frac{d\tau_{in,t}}{dt}+c_a\rho_{out}Q_a(\tau_{in,t}-\tau_{out}) = \frac{q_f-q_h}{60} \qquad (6)$$

The change in indoor temperature with time (in minutes) due to the operation of the appliance can be expressed as

$$\frac{d\tau_{in,t}}{dt} = \frac{\frac{q_f-q_h}{60}+c_a\rho_{out}Q_a(\tau_{out}-\tau_{in,t})}{c_a\rho_{in}V} \qquad (7)$$

Noting that $Q_a/V = \mathrm{ACH}/60 = \kappa_a$, and solving for the indoor temperature at time t gives

$$\begin{aligned}\tau_{in,t} = \{&\frac{q_f-q_h}{60}+c_a\rho_{out}Q_a\tau_{out} \\ &-\left[\frac{q_f-q_h}{60}+c_a\rho_{out}Q_a(\tau_{out}-\tau_{in,0})\right] \\ &\times\exp\left(-\kappa_a\frac{\rho_{out}}{\rho_{in}}t\right)\}/c_a\rho_{out}Q_a\end{aligned} \qquad (8)$$

Cycling Appliance Use

For the cycling case, several assumptions are required to simplify the model development. First, it is assumed that the appliance has a consistent cycle time, including both on and off periods. Second, it is assumed that the fraction of the cycle time that the appliance is on is also the same for every cycle.

The equations that follow assume that the calculation is started when the appliance first comes on at the beginning of the first cycle.

Concentration. In general, the equation governing the concentration change during on-periods is

$$\begin{aligned}C_{i,t} = [&K_{i,f}+K_{i,p}+\kappa_aC_{i,out} \\ &-(K_{i,f}+K_{i,p}+\kappa_aC_{i,out}-\kappa_aC_{i,0})\exp(-\kappa_at)]/\kappa_a\end{aligned} \qquad (9)$$

and the equation governing the concentration change during off-periods is

$$\begin{aligned}C_{i,t} = [&K_{i,p}+\kappa_aC_{i,out} \\ &-(K_{i,p}+\kappa_aC_{i,out}-\kappa_aC_{i,0})\exp(-\kappa_at)]/\kappa_a\end{aligned} \qquad (10)$$

Equation 10 is the same as Equation 9, except that it has the contribution of the appliance $K_{i,f}$ removed.

For each successive period during the cycling, the "initial" concentration $C_{i,0}$ is the final concentration of the preceding period.

In the first cycle, beginning with the unvented appliance coming on at time 0 and operating until time fT, the concentration of any gas i at any time $t < fT$ is

$$\begin{aligned}C_{i,t} = [&K_{i,f}+K_{i,p}+\kappa_aC_{i,out} \\ &-(K_{i,f}+K_{i,p}+\kappa_aC_{i,out}-\kappa_aC_{i,0}) \\ &\times\exp(-\kappa_at)]/\kappa_a\end{aligned} \qquad (11)$$

and the concentration of any gas i at the end of the on-portion of the cycle is

$$\begin{aligned}C_{i,fT} = [&K_{i,f}+K_{i,p}+\kappa_aC_{i,out} \\ &-(K_{i,f}+K_{i,p}+\kappa_aC_{i,out}-\kappa_aC_{i,0})\exp(-\kappa_afT)]/\kappa_a\end{aligned} \qquad (12)$$

For the off-portion of the first cycle, the integration equation is

$$\int_{C_{fT0}}^{C_{i,t}}\frac{dC_{i,t}}{K_{i,p}+\kappa_aC_{i,out}-\kappa_aC_{i,t}} = \int_{fT}^{t}dt \qquad (13)$$

Solving for the concentration of gas i at time $fT < t < T$ gives

$$\begin{aligned}C_{i,t} = \{&K_{i,p}+\kappa_aC_{i,out} \\ &-(K_{i,p}+\kappa_aC_{i,out}-\kappa_aC_{i,fT}) \\ &\times\exp[-\kappa_a(t-fT)]\}/\kappa_a\end{aligned} \qquad (14)$$

and the concentration of any gas i at the end of the cycle time T is

$$\begin{aligned}C_{i,T} = \{&K_{i,p}+\kappa_aC_{i,out} \\ &-(K_{i,p}+\kappa_aC_{i,out}-\kappa_aC_{i,fT}) \\ &\times\exp[-\kappa_a(1-f)T]\}/\kappa_a\end{aligned} \qquad (15)$$

Substituting Equation 12 for $C_{i,fT}$ and simplifying results in the following equation for the concentration at the end of the full cycle,

$$\begin{aligned}C_{i,T} = \{&K_{i,p}+\kappa_aC_{i,out}+K_{i,f}\exp[-\kappa_a(1-f)T] \\ &-(K_{i,f}+K_{i,p}+\kappa_aC_{i,out}-\kappa_aC_{i,0}) \\ &\times\exp(-\kappa_aT)\}/\kappa_a\end{aligned} \qquad (16)$$

Repeating this process through multiple cycles, a pattern emerges that allows for the determination of the gas concentration at any time t. The solution can be most easily developed as four separate pieces. These pieces include the equations for the concentration (1) at the end of any full cycle; (2) at the end of any appliance on-portion of a cycle; (3) at any time t during the appliance on-portion of the cycle; and (4) at any time t during the appliance off-portion of the cycle.

1. Equation for End of Full Cycle

The equation for the concentration of any gas i at the end of a full cycle N can be expressed as

$$\begin{aligned}C_{i,NT} = \{&K_{i,p}+\kappa_aC_{i,out}+K_{i,f} \\ &\times\left[\sum_{n=1}^{N}\exp(-\kappa_a(n-f)T)-\sum_{n=2}^{N}\exp(-\kappa_a(n-1)T)\right] \\ &-(K_{i,f}+K_{i,p}+\kappa_aC_{i,out}-\kappa_aC_{i,0}) \\ &\times\exp(-\kappa_aNT)\}/\kappa_a\end{aligned} \qquad (17)$$

2. **Equation for End of Appliance On-Portion of Cycle**

The equation for the concentration of any gas i at the end of the appliance on-portion of any cycle can be expressed as

$$\begin{aligned} C_{i,NT+fT} = {} & \{K_{i,p} + \kappa_a C_{i,out} + K_{i,f} \\ & \left[\sum_{n=0}^{N} \exp(-\kappa_a nT) - \sum_{n=1}^{N} \exp(-\kappa_a (n-1+f)T)\right] \\ & - (K_{i,f} + K_{i,p} + \kappa_a C_{i,out} - \kappa_a C_{i,0}) \\ & \times \exp(-\kappa_a (N+f)T)\}/\kappa_a \end{aligned} \tag{18}$$

where N is the number of full cycles preceding the appliance on-portion of interest.

3. **Equation for Time during Appliance On-Portion of Cycle**

The equation for the concentration of any gas i at any time t during the appliance on-portion of any cycle can be expressed as

$$\begin{aligned} C_{i,NT+p} = {} & \{K_{i,f} + K_{i,p} + \kappa_a C_{i,out} \\ & + K_{i,f}[\sum_{n=1}^{N} \exp{-\kappa_a}[(n-f)T+p] \\ & - \sum_{n=1}^{N} \exp(-\kappa_a[(n-1)T+p])\ ] \\ & - (K_{i,f} + K_{i,p} + \kappa_a C_{i,out} - \kappa_a C_{i,0}) \\ & \times \exp(-\kappa_a (NT+p))\}/\kappa_a \end{aligned} \tag{19}$$

where N is the number of full cycles preceding the appliance on-portion of interest and p is the number of time steps into the on-portion of the cycle.

Note that in this equation there is a term $K_{i,f}$ at the beginning, unlike the other equations in this set.

4. **Equation for Time during Appliance Off-Portion of Cycle**

The equation for the concentration of any gas i at any time t during the appliance off-portion of any cycle can be expressed as

$$\begin{aligned} C_{i,NT+fT+p} = {} & \{K_{i,p} + \kappa_a C_{i,out} + K_{i,f} \\ & \times \left[\sum_{n=0}^{N} \exp\left((-\kappa_a[nT+p]) - \sum_{n=1}^{N} \exp(-\kappa_a[(n-1+f)T+p])\right)\right] \\ & - (K_{i,f} + K_{i,p} + \kappa_a C_{i,out} - \kappa_a C_{i,0}) \exp(-\kappa_a((N+f)T+p))\}/\kappa_a \end{aligned} \tag{20}$$

where N is the number of full cycles preceding the appliance on-portion of interest and p is the number of time steps into the off-portion of interest.

Room Temperature. The equations for room temperature in the cycling case can be developed in the same fashion as those for the gas concentration. The resulting equations for the four pieces are

1. **Equation for end of full cycle**

The equation for the temperature at the end of a full cycle N can be expressed as

$$\begin{aligned} \tau_{in,NT} = {} & \{\frac{-q_h}{60} + c_a \rho_{out} Q_a \tau_{out} + \frac{q_f}{60} \\ & \times \left[\sum_{n=1}^{N} \exp\left(-\kappa_a \frac{\rho_{out}}{\rho_{in}}(n-f)T\right) - \sum_{n=2}^{N} \exp\left(-\kappa_a \frac{\rho_{out}}{\rho_{in}}(n-1)T\right)\right] \\ & - \left(\frac{q_f - q_h}{60} + c_a \rho_{out} Q_a (\tau_{out} - \tau_{in,0})\right) \exp\left(-\kappa_a \frac{\rho_{out}}{\rho_{in}} NT\right)\} \\ & \div c_a \rho_{out} Q_a \end{aligned} \tag{21}$$

2. **Equation for end of appliance on-portion of cycle**

The equation for the temperature at the end of the appliance on-portion of any cycle can be expressed as

$$\begin{aligned} \tau_{in,NT+fT} = {} & \{\frac{-q_h}{60} + c_a \rho_{out} Q_a \tau_{out} + \frac{q_f}{60} \\ & \times \left[\sum_{n=0}^{N} \exp\left(-\kappa_a \frac{\rho_{out}}{\rho_{in}} nT\right) - \sum_{n=1}^{N} \exp\left(-\kappa_a \frac{\rho_{out}}{\rho_{in}}(n-1+f)T\right)\right] \\ & - \left(\frac{q_f - q_h}{60} + c_a \rho_{out} Q_a (\tau_{out} - \tau_{in,0})\right) \exp\left(-\kappa_a \frac{\rho_{out}}{\rho_{in}}(N+f)T\right)\} \\ & \div c_a \rho_{out} Q_a \end{aligned} \tag{22}$$

where N is the number of full cycles preceding the appliance on-portion of interest.

3. **Equation for temperature during appliance on-portion of cycle**

The equation for the temperature at any time t during the appliance on-portion of any cycle can be expressed as

$$\begin{aligned} \tau_{in,NT+p} = {} & \{\frac{q_f - q_h}{60} + c_a \rho_{out} Q_a \tau_{out} + \frac{q_f}{60} \\ & \times [\sum_{n=1}^{N} \exp\left(-\kappa_a \frac{\rho_{out}}{\rho_{in}}[(n-f)T+p]\right) \\ & - \sum_{n=1}^{N} \exp\left(-\kappa_a \frac{\rho_{out}}{\rho_{in}}[(n-1)T+p]\right)] \\ & - \left(\frac{q_f - q_h}{60} + c_a \rho_{out} Q_a (\tau_{out} - \tau_{in,0})\right) \\ & \times \exp\left(-\kappa_a \frac{\rho_{out}}{\rho_{in}}(NT+p)\right)\}/c_a \rho_{out} Q_a \end{aligned} \tag{23}$$

where N is the number of full cycles preceding the appliance on-portion of interest and p is the number of time steps into the on-portion of the cycle.

Note that in this equation there is a term $q_f/60$ at the beginning, unlike the other equations in this set.

4. **Equation for temperature during appliance off-portion of cycle**

The equation for the temperature at any time t during the appliance off-portion of any cycle can be expressed as

Table 1. House Characteristics for Input into Model

Characteristic	Value	Units
Floor area	186	m^2
Ceiling height	2.44	m
Volume	453	m^3
Ventilation rate	0.35	ACH
Number of occupants	4	People
Fireplace heating capacity	11.137	kW
House heat loss rate	0.1465	kW/°C
Indoor temperature	22.2	°C
Outdoor temperature	1.67	°C
House heating load	10.852	kW
Indoor relative humidity	50	%
Outdoor relative humidity	50	%

$$\begin{aligned}\tau_{NT+fT+p} = \{ & \frac{-q_h}{60} + c_a \rho_{out} Q_a \tau_{out} + \frac{q_f}{60} \\ & \times \Big[\sum_{n=0}^{N} \exp\Big(-\kappa_a \frac{\rho_{out}}{\rho_{in}}[nT+p]\Big) \\ & - \sum_{n=1}^{N} \exp\Big(-\kappa_a \frac{\rho_{out}}{\rho_{in}}[(n-1+f)T+p]\Big)\Big] \\ & - \Big(\frac{q_f - q_h}{60} + c_a \rho_{out} Q_a (\tau_{out} - \tau_{in,0})\Big) \\ & \times \exp\Big(-\kappa_a \frac{\rho_{out}}{\rho_{in}}((N+f)T+p)\Big)\} / c_a \rho_{out} Q_a \end{aligned} \tag{24}$$

where N is the number of full cycles preceding the appliance on-portion of interest and p is the number of time steps into the off-portion of interest.

EXAMPLE

The following example assumes that about half of the heating load is met using another heat source and that the fireplace is on for 5 minutes of every 10-minute cycle.

Consider a house with the assumed characteristics, unvented fireplace heating capacity, and outdoor conditions shown in Table 1. The house heat loss rate was selected to provide a heating load at the listed outdoor temperature similar to the fireplace heating capacity, and is comparable to a home with uninsulated walls, a slab-on-grade floor, and about 1.6 cm of fiberglass insulation. The constant ventilation rate could be approximately achieved by a tight house with constant mechanical ventilation, or could reflect a natural infiltration rate estimated using tracer gases or a blower door combined with an infiltration model. If the ventilation rate were based on infiltration, it would not remain constant for very long; however, over a limited time the rate may be approximately constant, and for a more full treatment the model could be incorporated into an hourly model that can estimate the infiltration rate continuously.

Table 2. Assumed Combustion Gas Emission Rates

Gas	Fireplace Emission, kg/(s·kW)	People Emission, kg/s·person	Outdoor Level, kg/m^3
Carbon monoxide	1.854e-04	0	0
Carbon dioxide	1.746e-01	3.277e-02	6.965e-04
Nitrogen dioxide	3.046e-05	0	0
Oxygen	–2.448e-01	–2.593e-02	2.781e-01
Water	1.303e-01	1.134e-01	3.931e-03

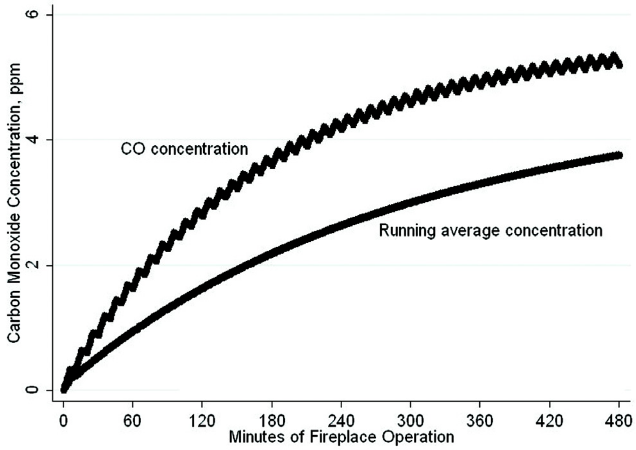
Next, assume that the emission rate of the fireplace, emission rate of the people, and the outdoor concentrations for each gas are those given in Table 2. The fireplace emission rates for carbon monoxide and nitrogen dioxide are the maximum levels allowed by ANSI Standard Z21.11.2.

Figure 1 shows the results for the five gases and temperature for 8 h of cycling unvented fireplace use. The time period of 8 h was selected to show numerous cycles and the curvature of the concentration accumulation, while still showing the cycling nature of the results. For carbon monoxide, carbon dioxide, and nitrogen dioxide (Figures 1A–1C, respectively), the running average is also shown. For those three gases and oxygen (Figure 1D), the threshold value(s) is shown for comparison. In general, the change in concentrations of gases is slightly more than half of the change as there would be for the same house and fireplace if used continuously.

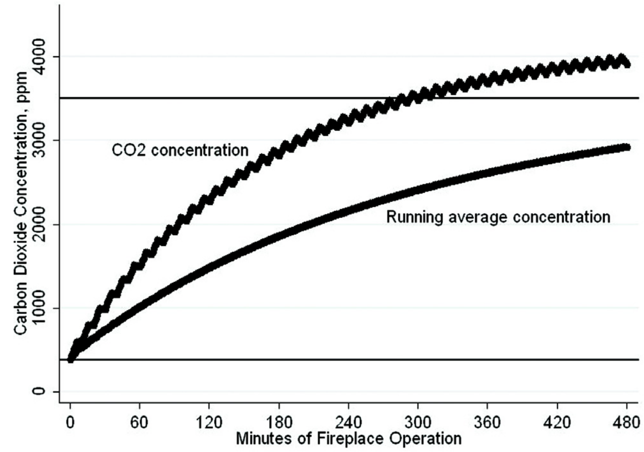
For carbon monoxide (Figure 1A), the 8-h average value of 9 ppm, based on the U.S. EPA health-based outdoor standard (US EPA 2000), is never exceeded during the 8 h period modeled, either based on the 1-minute readings or based on the running average. The maximum concentration after 8 h of use is below 6 ppm.

For carbon dioxide (Figure 1B), the Health Canada long-term average guideline of 3500 ppm (Health Canada 1995) is never exceeded based on the running average, and the 1-minute concentration exceeds 3500 ppm at about 5 h. This is more than 3 times the length of time that it would have taken for the 1-minute values to exceed the 3500 ppm level under continuous use. These results indicate that eventually the long-term average will exceed 3500 ppm if the fireplace continues to be used in this manner, but it will take considerably longer than 8 h.

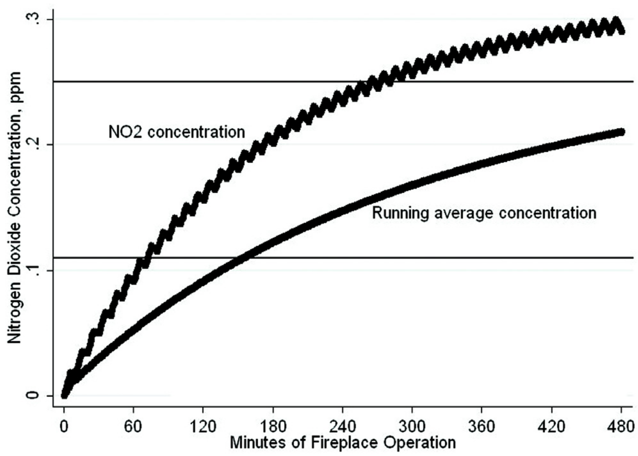
Nitrogen dioxide (Figure 1C) exceeds both 1-hour threshold levels (110 from the World Health Organization [WHO 2000] and 250 ppb from Health Canada [1995]) in a shorter time frame than any other gas. The running average exceeds



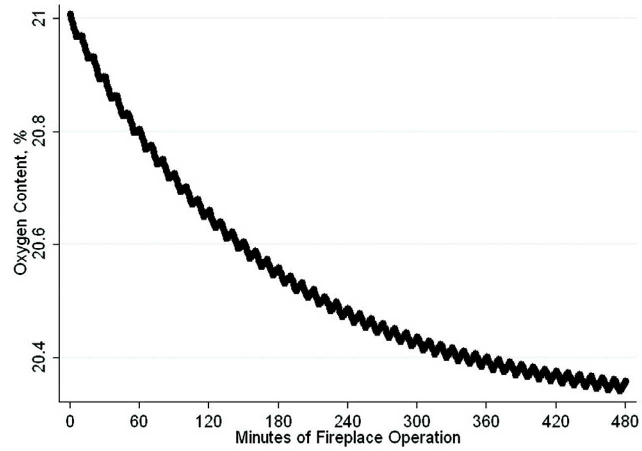
(A) Carbon Monoxide



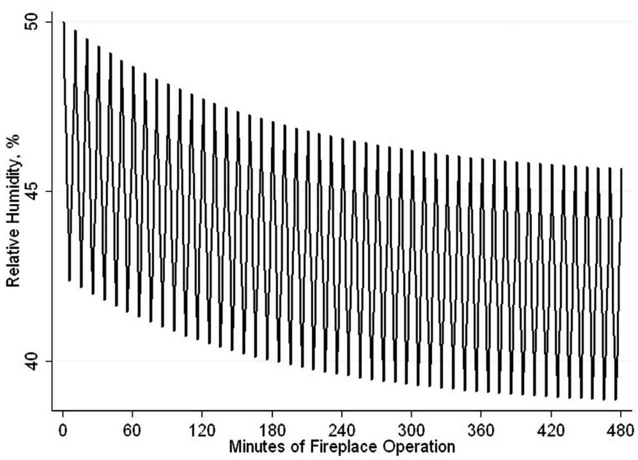
(B) Carbon Dioxide



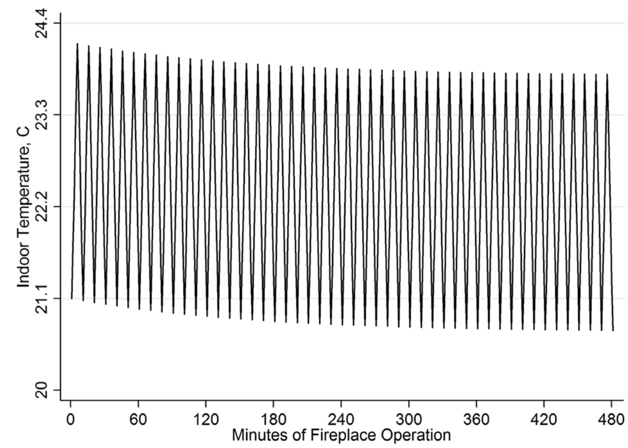
(C) Nitrogen Dioxide



(D) Oxygen



(E) Relative Humidity



(F) Temperature

Figure 1 *Indoor gas concentrations, relative humidity, and temperature for cycling use using the assumptions from Tables 1 and 2.*

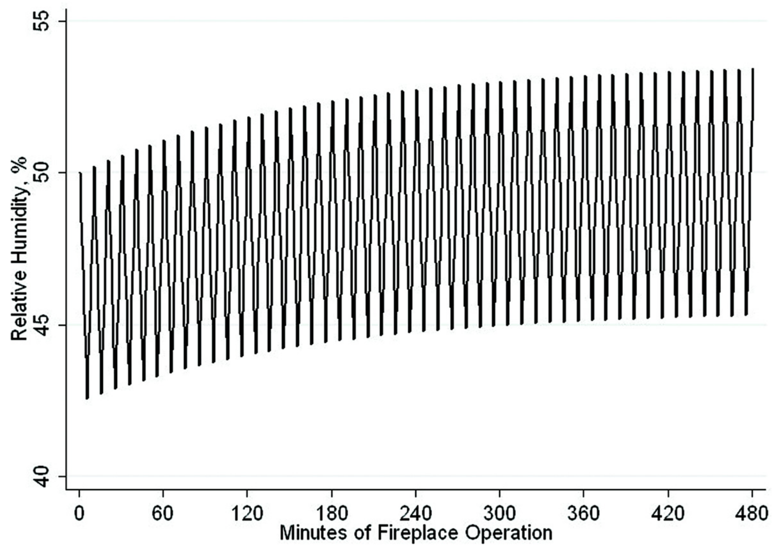


Figure 2 *Relative humidity for the cycling case assuming that outdoor relative humidity is 80%.*

the 110 ppb level in about 3 h but does not exceed the 250 ppb level within the 8-h time period. One-minute concentrations exceed the 110 ppb level in about 1 h, and the 250 ppb level in about 4.5 h. These results suggest that nitrogen dioxide is the gas most likely to exceed the threshold values, which agrees with the field results from the project.

Within the 8-h period modeled, the oxygen content in the home (Figure 1D) did not drop below the NIOSH definition of an oxygen-depleted environment, 19.5% (NIOSH 1979). After 8 hours the level dropped to about 20.4%.

The relative humidity actually tends to decrease over time (Figure 1E). This is because the amount of water put into the home by the fireplace is less than the drying that results from bringing in the outdoor air, because the fireplace contribution is intermittent while the outdoor air drying is continuous.

The temperature in the house (Figure 1F) oscillates by about 2°F around the set point of 22.2°F using these assumptions. This is consistent with the intended result from using the fireplace in a cycling mode controlled by a thermostat.

It is reasonable to question what would happen if the outdoor humidity were higher, such that the drying potential from infiltration was not as high. That was investigated for the cycling case by assuming that the outdoor air humidity was 80% instead of 50%. The humidity results for this case are shown in Figure 2. This graph shows that the indoor humidity does in fact have an increasing trend, but that even after 8 hours the relative humidity is still below 55%.

DISCUSSION

We have developed a model that provides a closed-form solution for indoor pollutant concentrations from a cycling source such as an unvented combustion appliance operating on a thermostat. This model allows for the calculation to be done at any time within any cycle, and because it is closed-form instead of time-series it is not necessary to calculate the concentrations at previous times.

The model does assume that the following remain constant: indoor air density, air change rate of the house, and cycle length and fractional on-time for the appliance. It also assumes an even distribution of combustion gases throughout the volume of the space, whereas in practice the concentrations outside of the immediate vicinity of the fireplace are typically about 70–80% of the level near the fireplace (Francisco et al. 2010). While these assumptions mean that the results will not be exactly correct, the model is useful for determining the most important factors in the resulting concentrations and which pollutants are likely to exceed thresholds first, and can help evaluate what can be done to prevent such exceedence, such as increasing the ventilation rate or limiting use of the appliance.

When applied to unvented gas fireplaces, the results of the model are in agreement with field testing that showed that nitrogen dioxide was the gas most likely to exceed published thresholds but that carbon monoxide was much less likely to exceed these values. The model results also are in agreement with field measurements in showing that the relative humidity in the room with the fireplace often stays in reasonable levels because of the influence of temperature on relative humidity. In locations more remote from the fireplace this often will not hold true.

The example provided simply shows how the model can be used. Other assumptions (e.g., different cycle lengths, use at colder outdoor temperatures, different emission rates, different air change rates, different house heat loss rates) can produce different results. It is the point of the model to be able to enter any set of inputs to estimate the results under those conditions.

NOMENCLATURE

$C_{i,0}$ = initial concentration of gas i
$C_{i,t}$ = concentration of gas i at time t
$C_{i,out}$ = outdoor concentration of gas i
t = time, min
f = fractional on-time for cycling case
T = cycle length, on plus off periods (min)
N = number of full cycles
p = number of time steps into appliance on- or off-portion of cycle, min
ACH = ventilation rate, air changes per hour 1/h
$C_{i,crit}$ = threshold concentration of gas i
$C_{i,ss}$ = steady-state concentration of gas i
t_{crit} = duration of continuous appliance use until $C_{i,crit}$ is exceeded, min
ACH_{crit} = ventilation rate to prevent concentration from ever exceeding $C_{i,crit}$, air changes per hour, 1/h
$E_{i,f}$ = emission rate of gas i from fireplace combustion, per unit heating capacity
$E_{i,p}$ = emission rate of gas i from occupants, per person
$K_{i,f}$ = total emission rate of gas i from fireplace combustion, per unit volume of indoor space
$K_{i,p}$ = total emission rate of gas i from occupants, per unit volume of indoor space
q_f = heating capacity of fireplace
q_h = heat loss of home excluding infiltration
N_p = number of people in home
V = volume of indoor space
κ_a = dilution rate in consistent units; = ACH/60 (1/min)
$\tau_{in,0}$ = initial indoor temperature
τ_{out} = outdoor temperature
$\tau_{in,t}$ = indoor temperature at time t
ρ_{in} = density of indoor air
ρ_{out} = density of outdoor air
c_a = specific heat of air
Q_a = infiltration/ventilation rate

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